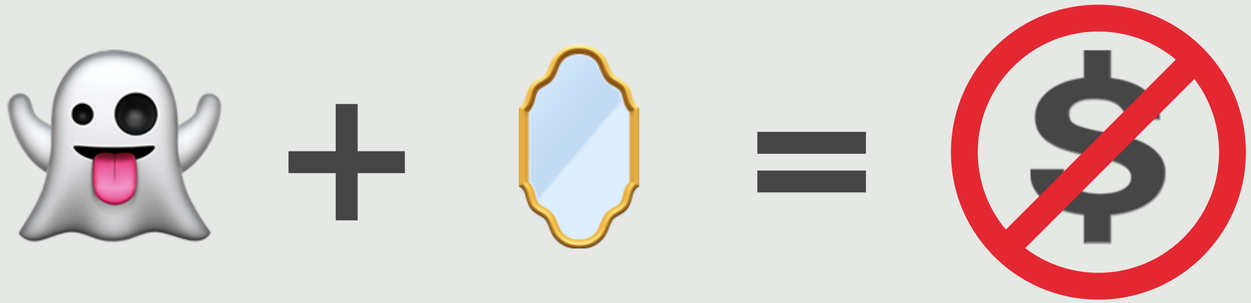
chocola

Dependent Ghosts have a reflection for free



Théo Winterhalter

What's up with vectors?

```
Inductive vec A : ℕ → Type :=
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| vcons (a : A) n (v : vec A n) : vec A (S n)
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type-based invariant

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`vec A (S k + m)` vs `vec A (k + S m)`

but we really wish they would be equal...

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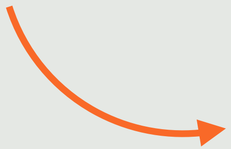

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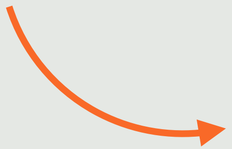
we should have lists!



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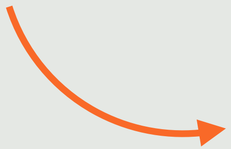

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The problem is always in the n of `vec A n`...

Using ghost types…

```
Inductive vec A : erased ℕ → Type :=
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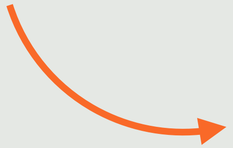
we mark the index as erased



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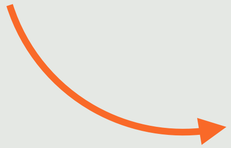

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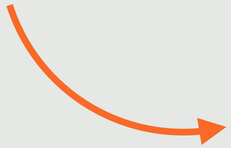

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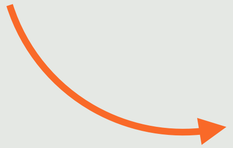

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gS : erased ℕ → erased ℕ
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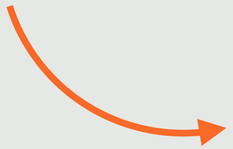

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gS : erased ℕ → erased ℕ
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but…

```
erased bool → bool
```

only contains constant functions

...and ghost reflection

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Eliminator `reveal` cannot land in `Type`
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$$\frac{A : \mathsf{Ghost} \qquad u, v : A \qquad e : u = v}{u \equiv v}$$

Propositionally equal inhabitants of ghosts
are definitionally equal

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ok because `vec A (gS k +' m) ≡ vec A (k +' gS m)`

Wait, couldn't this just be (S)Prop?

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this is a problem though!

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Reveal proposition

$$\frac{e : \texttt{erased}\ A \qquad f : A \to \texttt{Prop}}{\texttt{Reveal}\ e\ f : \texttt{Prop}}$$

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We get a discriminator: `D (hide 0) ↔ ⊤` `D (gS n) ↔ ⊥`

```
D : erased ℕ → Prop
D n := Reveal n (λx. match x with 0 => ⊤ | _ => ⊥ end)
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translating *derivations*
replacing conversion rule by casts
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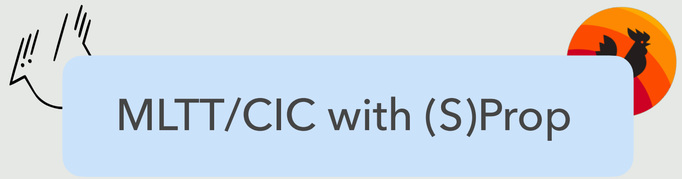

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parametricity translation
taking inspiration from exceptional type theory
[Pédrot Tabareau 2018]

MLTT/CIC with (S)Prop

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interesting on its own!

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Eliminating reflection

General case: Extenstional and intensional type theory

ETT = ITT + $\dfrac{e : u = v}{u \equiv v}$ 

Equality reflection

Propositionally equal **terms**

are definitionally equal

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 Key idea for ETT → ITT

Use transports everywhere
equality reflection is needed

Eliminating reflection

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We want $[\![.]\!]$ such that

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ITT

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Let's look at the **conversion** rule

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$$\dashrightarrow$$

$$[\![\Gamma]\!] \vdash_i [\![t]\!] : [\![A]\!]$$

$$[\![\Gamma]\!] \vdash_i p : [\![A]\!] = [\![B]\!]$$

so $[\![\Gamma]\!] \vdash_i \mathtt{transp}\ p\ [\![t]\!] : [\![B]\!]$

Eliminating reflection

General case: ETT → ITT

We want $[\![.]\!]$ such that

ETT

$$\Gamma \vdash_x t : A \quad \text{implies} \quad [\![\Gamma]\!] \vdash_i [\![t]\!] : [\![A]\!]$$

$$\Gamma \vdash_x u \equiv v : A \quad \text{implies} \quad [\![\Gamma]\!] \vdash_i p : [\![u]\!] =_{[\![A]\!]} [\![v]\!]$$

ITT

Let's look at the **conversion** rule

$$\frac{\Gamma \vdash_x t : A \qquad \Gamma \vdash_x A \equiv B}{\Gamma \vdash_x t : B}$$

$$[\![\Gamma]\!] \vdash_i [\![t]\!] : [\![A]\!]$$

$$[\![\Gamma]\!] \vdash_i p : [\![A]\!] = [\![B]\!]$$

so $[\![\Gamma]\!] \vdash_i \mathtt{transp}\ p\ [\![t]\!] : [\![B]\!]$

We wanted $[\![t]\!]$!

Eliminating reflection

General case: ETT → ITT

We want $[\![.]\!]$ such that

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ITT

$[\![t]\!]$ should be a *class of terms* with

$$t' \in [\![t]\!] \quad \text{implies} \quad \texttt{transp}\ p\ t' \in [\![t]\!]$$

Eliminating reflection

General case: ETT → ITT

We want $[\![.]\!]$ such that

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$$\Gamma \vdash_x t : A \quad \text{implies} \quad \Gamma' \vdash_i t' : A'$$

$$\Gamma \vdash_x u \equiv v : A \quad \text{implies} \quad \Gamma' \vdash_i p : u' =_{A'} v'$$

ITT

for some $\Gamma' \in [\![\Gamma]\!]$, $A' \in [\![A]\!]$, …

$[\![t]\!]$ should be a *class of terms* with

$$t' \in [\![t]\!] \quad \text{implies} \quad \mathtt{transp}\ p\ t' \in [\![t]\!]$$

Eliminating reflection

General case: ETT → ITT

We want $[\![.]\!]$ such that

ETT

$\Gamma \vdash_{\mathbf{x}} t : A$ implies $\Gamma' \vdash_{\mathbf{i}} t' : A'$

$\Gamma \vdash_{\mathbf{x}} u \equiv v : A$ implies $\Gamma' \vdash_{\mathbf{i}} p : u' =_{A'} v'$

ITT

for some $\Gamma' \in [\![\Gamma]\!]$, $A' \in [\![A]\!]$, …

conversion rule again:

$$\frac{\Gamma \vdash_x t : A \qquad \Gamma \vdash_x A \equiv B}{\Gamma \vdash_x t : B}$$

⇢

$\Gamma' \vdash_i t' : A'$

$\Gamma'' \vdash_i p : A'' =_T B'$

Eliminating reflection

General case: ETT → ITT

We want $[\![.]\!]$ such that

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$\Gamma \vdash_x t : A$ implies $[\![\Gamma]\!] \vdash_i [\![t]\!] : [\![A]\!]$

$\Gamma \vdash_x u \equiv v : A$ implies $[\![\Gamma]\!] \vdash_i p : [\![u]\!] =_{[\![A]\!]} [\![v]\!]$

ITT

for some $[\![\Gamma]\!] \in [\![\Gamma]\!]$, $[\![A]\!] \in [\![A]\!]$, …

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$$\frac{\Gamma \vdash_x t : A \qquad \Gamma \vdash_x A \equiv B}{\Gamma \vdash_x t : B}$$

$\Rightarrow$

$$\Gamma' \vdash_i t' : A'$$

$$\Gamma'' \vdash_i p : A'' =_{T'} B'$$

$T' \in [\![\mathrm{Type}]\!]$

Eliminating reflection

General case: ETT → ITT

We want $[\![.]\!]$ such that

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ITT

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$\Gamma' \vdash_i t' : A'$

$\Gamma'' \vdash_i p : A'' =_{T'} B'$

$T' \in [\![\mathsf{Type}]\!]$

we need to relate two translations of the same object (at possibly two different types)

Eliminating reflection

General case: ETT → ITT

We want $[\![.]\!]$ such that

ETT

$$\Gamma \vdash_x t : A \quad \text{implies} \quad \Gamma' \vdash_i t' : A'$$

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ITT

for some $\Gamma' \in [\![\Gamma]\!], A' \in [\![A]\!], \ldots$

Fundamental lemma

If $t', t'' \in [\![t]\!]$ then
t' and t'' are heterogeneously equal

Eliminating reflection

General case: ETT → ITT

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ITT

for some $\Gamma' \in [\![\Gamma]\!], A' \in [\![A]\!], \ldots$

Fundamental lemma

If $t', t'' \in [\![t]\!]$ then
t' and t'' are heterogeneously equal

Then we need UIP and FunExt...
and we end up with enormous terms!

Eliminating reflection

Everything is better with ghosts!

GRTT

$$\frac{A : \mathsf{Ghost} \qquad u, v : A \qquad e : u = v}{u \equiv v}$$

Ghost reflection

GTT

$$\frac{A : \mathsf{Ghost} \qquad e : u =_A v \qquad t : P\ u}{\mathsf{cast}\ e\ P\ t : P\ v}$$

$$\mathsf{cast}\ e\ P\ t \equiv t$$

ignored for conversion

Ghost casts

Eliminating reflection

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Ghost reflection

GTT

$$\frac{A : \mathsf{Ghost} \qquad e : u =_A v \qquad t : P\ u}{\mathsf{cast}\ e\ P\ t : P\ v}$$

Ghost casts

Two translations of the same term only differ in casts and are thus definitionally equal

$$\mathsf{cast}\ e\ P\ t \equiv t$$

ignored for conversion

Eliminating reflection

Everything is better with ghosts!

We define ⟦.⟧ a class of terms such that

GRTT

$$\Gamma \vdash_x t : A \quad \text{implies} \quad \Gamma' \vdash_g t' : A'$$

$$\Gamma \vdash_x u \equiv v : A \quad \text{implies}^* \quad \Gamma' \vdash_g u' \equiv v' : A'$$

GTT

for some $\Gamma' \in \llbracket \Gamma \rrbracket$, $A' \in \llbracket A \rrbracket$, …

Eliminating reflection

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for some $\Gamma' \in \llbracket \Gamma \rrbracket$, $A' \in \llbracket A \rrbracket$, …

But there's a catch! Ghost reflection only applies at the top-level.

$$\frac{A : \mathsf{Ghost} \qquad u, v : A \qquad e : u = v \qquad t : P\ u}{t : P\ v}$$

Eliminating reflection

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$$\Gamma \vdash_x t : A \quad \text{implies} \quad \Gamma' \vdash_g t' : A'$$

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for some $\Gamma' \in \llbracket \Gamma \rrbracket$, $A' \in \llbracket A \rrbracket$, …

But there's a catch! Ghost reflection only applies at the top-level.

$$\frac{A : \mathsf{Ghost} \qquad u, v : A \qquad e : u = v \qquad t : P\ u}{t : P\ v}$$

In other words, no going under binders…
Conjecture. Assuming FunExt for ghosts should be enough.

Consequences of the translation

Conservativity

If $\vdash_x$ `A : Type` and $\vdash_x$ `t : A`

then there exists $\vdash_g$ `t' : A` with `t' ∈ ⟦t⟧`

Consequences of the translation

Conservativity

If $\vdash_x$ `A : Type` and $\vdash_x$ `t : A`

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Proof using the fact that `A` is definitionally equal to any of its translations `A'`

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Relative consistency

If $\vdash_x$ `t : ⊥`

then there exists $\vdash_g$ `t' : ⊥`

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If $\vdash_x$ `A : Type` and $\vdash_x$ `t : A`

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Relative consistency

If $\vdash_x$ `t : ⊥`

then there exists $\vdash_g$ `t' : ⊥`

Proof using conservativity

How do we justify this?

$$\frac{A : \mathsf{Ghost} \qquad u, v : A \qquad e : u = v}{u \equiv v}$$

Ghost reflection

translating *derivations*
replacing conversion rule by casts
like for ETT to ITT [Oury 2005 ; WST 2019]

How do we justify this?

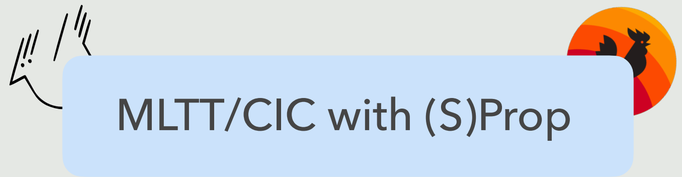
$$\frac{A : \mathsf{Ghost} \qquad e : u =_A v \qquad t : P\ u}{\mathtt{cast}\ e\ P\ t : P\ v}$$

$$\mathtt{cast}\ e\ P\ t \equiv t$$

Ghost casts

ignored for conversion

parametricity translation
taking inspiration from exceptional type theory
[Pédrot Tabareau 2018]



Erasure

Translation getting rid of all ghosts and proofs

$$[\lambda(x^{\mathbb{T}} : A).\ t]_\varepsilon := \lambda(x : [\![A]\!]_\varepsilon).\ [t]_\varepsilon$$

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$$[f^{\mathbb{T}}\ u^{\mathbb{T}}]_\varepsilon := [f]_\varepsilon\ [u]_\varepsilon$$

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$\mathtt{exfalso}^{\mathbb{T}}\ (A : \mathtt{Type})\ (p : \bot) : A$

$$[\mathtt{exfalso}^{\mathbb{T}}\ A\ p]_\varepsilon := ??$$

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we get no $\bot$ but we need some $[\![A]\!]_\varepsilon$

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$\mathtt{exfalso}^{\mathbb{T}}\ (A : \mathtt{Type})\ (p : \bot) : A$

$$[\mathtt{exfalso}^{\mathbb{T}}\ A\ p]_\varepsilon := [\![A]\!]_\varnothing$$

we get no $\bot$ but we need some $[\![A]\!]_\varepsilon$

$$A : \mathtt{Type} \longrightarrow \begin{array}{l} [\![A]\!]_\varepsilon : \mathtt{Type} \\ [\![A]\!]_\varnothing : [\![A]\!]_\varepsilon \end{array}$$

Example

Booleans

source

```
Inductive bool :=
| true
| false
```

Example

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erasure

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parametricity in `Prop` [Keller Lasson 2012]

```
Inductive boolP : bool• → Prop :=
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predicate guaranteeing no exceptions raised at top-level

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⚠ limits large elimination

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Free theorem:

```
erased bool → bool
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only contains constant functions

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```
∀ (f : erased bool → bool) (P : bool → Prop),
  P (f (hide true)) → P (f (hide false))
```

Example

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is justified in the model by

```
∀ (fe : bool•) (fP : ∀ b, boolP b → boolP fe)
  (P : bool• → unit) (PP : ∀ b, boolP b → Prop),
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proof irrelevance

Example

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```

the key is that it is erased to a constant

proof irrelevance

Vectors

```
Inductive vec A : erased ℕ → Type :=
| vnil : vec A (hide 0)
| vcons (a : A) n (v : vec A n) : vec A (gS n)
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Inductive vec• (A : ty) :=
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```

```
⟦vec A n⟧P := vecP [A]ε ⟦A⟧P ⟦n⟧v ⟦n⟧P
```

we need **revival** to recover n

Vectors

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```

we need **revival** to recover n

Computation for the **eliminator**

```
vec-elim vnil P z s ≡ z
```

Unusual

Vectors

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Inductive vec A : erased ℕ → Type :=
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| vcons (a : A) n (v : vec A n) : vec A (gS n)
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```

```
⟦vec A n⟧P := vecP [A]ε ⟦A⟧P ⟦n⟧v ⟦n⟧P
```

we need **revival** to recover n

Computation for the **eliminator**

```
vec-elim vnil P z s ≡ z
```

```
vec-elim (vcons a n v) P z s ≡ s a (glength v) v (vec-elim v P z s)
```

Revival

Revival essentially gets back what was erased

If $\Gamma \vdash t : A$ and $\Gamma \vdash t :: \mathbb{G}$ in GTT
then $[\![\Gamma]\!]_P \vdash [\![t]\!]_V : [\![A]\!]_\varepsilon$ in CC

(undefined otherwise)

Revival

Revival essentially gets back what was erased

If $\Gamma \vdash t : A$ and $\Gamma \vdash t :: \mathbb{G}$ in GTT
then $[\![\Gamma]\!]_P \vdash [\![t]\!]_V : [\![A]\!]_\varepsilon$ in CC

(undefined otherwise)

Here we use modes: $\mathbb{T}$, $\mathbb{G}$, and $\mathbb{P}$ that are syntactically determined

$\Gamma \vdash t :: \mathbb{G}$ approximates
$\Gamma \vdash t : A :$ `Ghost` for some A

Revival

Revival essentially gets back what was erased

If $\Gamma \vdash t : A$ and $\Gamma \vdash t :: \mathbb{G}$ in GTT
then $[\![\Gamma]\!]_{\mathbb{P}} \vdash [\![t]\!]_{\mathbb{V}} : [\![A]\!]_{\varepsilon}$ in CC

(undefined otherwise)

Here we use modes: $\mathbb{T}$, $\mathbb{G}$, and $\mathbb{P}$ that are syntactically determined

$\Gamma \vdash t :: \mathbb{G}$ approximates
$\Gamma \vdash t : A :$ `Ghost` for some A

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Propositions essentially preserved (including $\bot$)
Hence consistency

Meta-theory

Joint work with Ewen Broudin--Caradec

Injectivity of Π types

If `Γ ⊢ ∀ (x : A). B ≡ ∀ (x : A'). B'`

then `Γ ⊢ A ≡ A'` and `Γ, x : A ⊢ B ≡ B'`

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Mode coherence

If `Γ ⊢ t : A` and `Γ ⊢ A : Sort m`

then `Γ ⊢ t :: m`

where

```
Sort 𝕋 = Type
Sort 𝔾 = Ghost
Sort ℙ = Prop
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Proof by validity (if `Γ ⊢ t : A` then `Γ ⊢ A : Sort m` for some `m` with `Γ ⊢ t :: m`) and uniqueness of type together with injectivity of sorts (proven like for Π types)

Perspectives

Overcome the need for parametricity in Prop

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general large elimination

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Better understand the theory for inductive types	fix (or understand better) the computation rule for `vec-elim`

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Internalise free theorems

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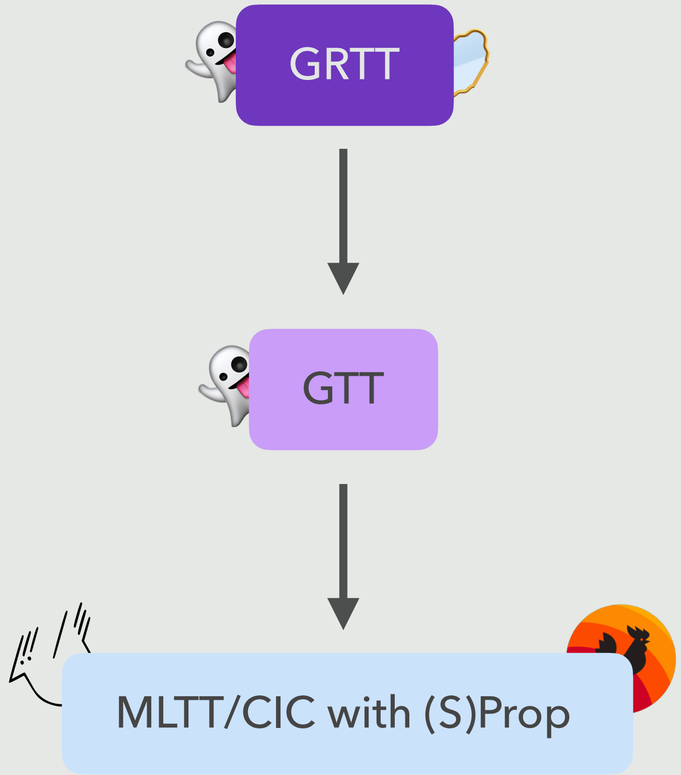
Decidability of typing

Integration in Coq and MetaCoq

Conclusion



Conclusion

Meta-theory

conservativity
consistency
type former discrimination
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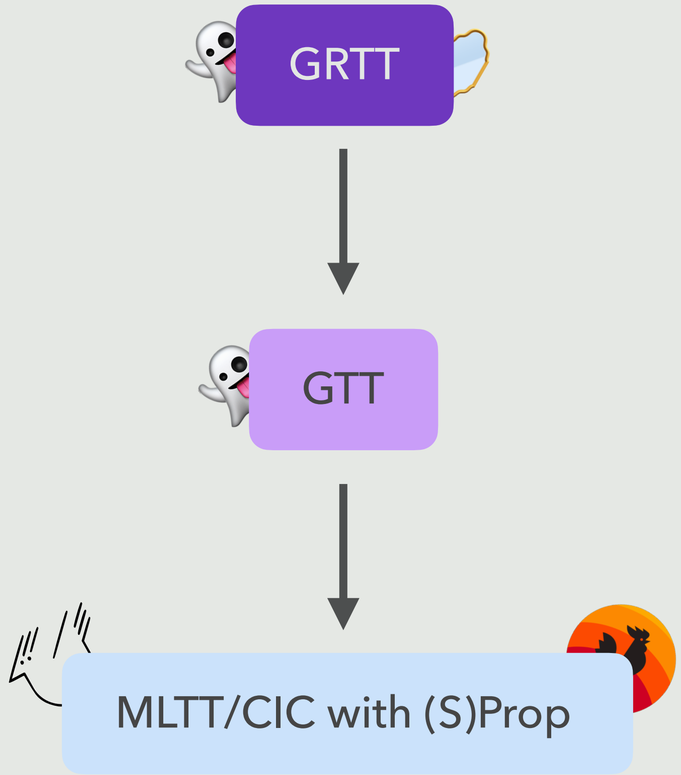

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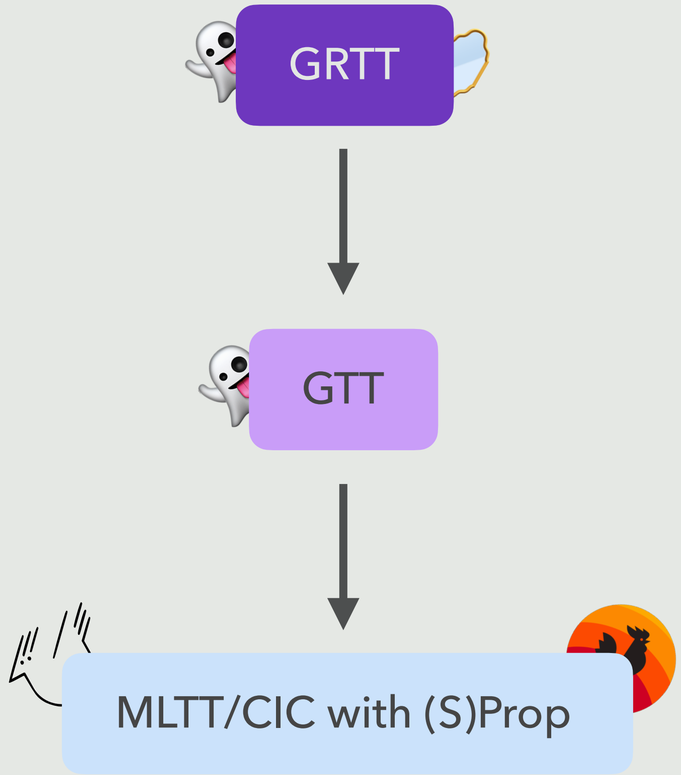

Formalisation in Coq

from scratch using Autosubst 2
improving its automation

trick to handle contexts of varying size

/TheoWinterhalter/ghost-reflection